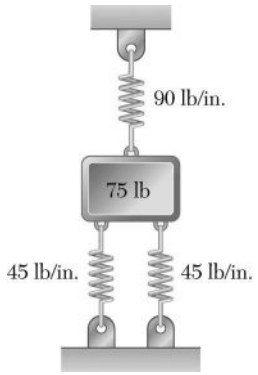


PROBLEM 19.18

A 75-lb block is supported by the spring arrangement shown. The block is moved vertically downward from its equilibrium position and released. Knowing that the amplitude of the resulting motion is 2 in., determine (*a*) the period and frequency of the motion, (*b*) the maximum velocity and maximum acceleration of the block.

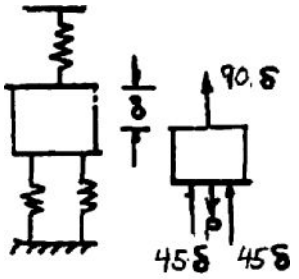
– یک بلوک ۷۵ پوندی مطابق شکل توسط فنرهایی نگه داشته می‌شود. بلوک در راستای قائم از وضعیت تعادل اولیه‌اش به سمت پایین کشیده و سپس رها می گردد. با علم به اینکه دامنه ارتعاش ۲ اینچ می باشد، تعیین نمایید:

الف: پریود (دوره تناوب) و فرکانس (بسامد) ارتعاش.

ب: بیشینه (ماکزیمم) سرعت و بیشینه شتاب بلوک.

SOLUTION

(*a*) Determine the constant k of a single spring equivalent to the three springs



$$P = k\delta$$

$$k\delta = 90\delta + 45\delta + 45\delta$$

$$k = 180 \text{ lb/in.} = 2160 \text{ lb/ft}$$

Natural frequency.

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{2160 \text{ lb/ft}}{\frac{75 \text{ lb}}{32.2 \text{ ft/s}^2}}}$$

$$\omega_n = 30.453 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{30.453} = 0.20633 \text{ s} \qquad \tau_n = 0.206 \text{ s} \blacktriangleleft$$

$$f_n = \frac{1}{\tau_n} \qquad f_n = 4.85 \text{ Hz} \blacktriangleleft$$

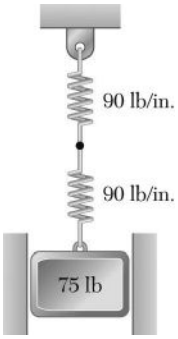
(*b*) $x = x_m \sin(\omega_n t + \phi) \qquad x_0 = 2 \text{ in.} = 0.16667 \text{ ft} = x_m$

$$\omega_n = 30.453 \text{ rad/s}$$

$$x = 0.16667 \sin(30.453t + \phi)$$

$$\dot{x} = (0.16667)(30.453)\cos(30.453t + \phi) \qquad v_{\max} = 5.08 \text{ ft/s} \blacktriangleleft$$

$$\ddot{x} = -(0.16667)(30.453)^2 \sin(30.453t + \phi) \qquad a_{\max} = 154.6 \text{ ft/s}^2 \blacktriangleleft$$



PROBLEM 19.19

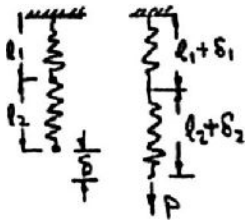
A 75-lb block is supported by the spring arrangement shown. The block is moved vertically downward from its equilibrium position and released. Knowing that the amplitude of the resulting motion is 2 in., determine (*a*) the period and frequency of the motion, (*b*) the maximum velocity and maximum acceleration of the block.

SOLUTION

(*a*) Determine the constant k of a single spring equivalent to the two springs shown.

$$\delta = \delta_1 + \delta_2 = \frac{P}{90 \text{ lb/in.}} + \frac{P}{90 \text{ lb/in.}} = \frac{P}{k}$$

$$\frac{1}{k} = \frac{1}{90} + \frac{1}{90} \qquad k = 45 \text{ lb/in.} = 540 \text{ lb/ft}$$



Period of the motion.

$$\tau_n = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{540}{75/32.2}}} = 0.41265 \text{ s} \qquad \tau_n = 0.413 \text{ s} \blacktriangleleft$$

$$f_n = \frac{1}{\tau_n} = \frac{1}{0.41265} = 2.42 \text{ Hz} \blacktriangleleft$$

(*b*)

$$x = x_m \sin(\omega_n t + \phi) \quad x_0 = 2 \text{ in.} = 0.16667 \text{ ft} = x_m$$

$$\omega_n = 2\pi f_n = 2\pi(2.4233) = 15.226 \text{ rad/s}$$

$$x = 0.16667 \sin(15.226t + \phi)$$

$$\dot{x} = (0.16667)(15.226)\cos(15.226t + \phi)$$

$$v_{\max} = 2.54 \text{ ft/s} \blacktriangleleft$$

$$\ddot{x} = -(0.16667)(15.226)^2 \sin(15.226t + \phi) \qquad a_{\max} = 38.6 \text{ ft/s}^2 \blacktriangleleft$$